

Further comments on nuclear forces in the chiral limit

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Abstract

We substantiate our statement that the deuteron remains bound in the chiral limit. We critically discuss recent claims that effective field theory cannot give a definite answer to this question.

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1. The question of the quark mass dependence of the nuclear forces has attracted considerable interest recently. The tool to perform such investigations is chiral effective field theory (EFT), which allows for a unified description of pion–nucleon and nucleon–nucleon dynamics. In [1] we had shown that the deuteron remains bound in the chiral limit, making use of an EFT approach based on a modified Weinberg power counting which successfully describes two-, three- and four-nucleon systems. In earlier [2] and parallel work [3] a different claim was made, namely that within the EFT one presently cannot make a definite statement whether or not the deuteron is bound in the chiral limit because some low-energy constants (LECs) are not known. In these works, a coordinate space power counting was used which is supposedly equivalent to the one employed by us. In a more recent paper, Beane and Savage [5] tried to substantiate their claim by performing an error analysis for the various input parameters similar to what was done already in [1]. Here, we wish to critically examine various claims made in that paper, and as a consequence, we are able to *substantiate our original statement*.

2. Since we do not wish to repeat any of the detailed arguments underlying our calculation presented in [1], we rather concentrate here on some issues raised in [5] which deserve a closer look.

- 1) In Ref. [5], the naive dimensional analysis for the quark (pion) mass dependent operators $\bar{D}_I M_\pi^2$ (where I specifies the channel under consideration) is repeated in terms of a parameter η . It is claimed that we use $\eta \leq 1/19$ in the 3S_1 channel. We are puzzled by such a statement. Even though the four-nucleon LECs are to some extent regulator and convention dependent, such effects should largely cancel in ratios of the LECs projected onto partial waves. So what we have really used for this relative strength is [1]:

$$\eta_{1S_0} \equiv \frac{\bar{D}_{1S_0} M_\pi^2}{C_0^{1S_0}} \leq \frac{1}{4.2}, \quad \eta_{3S_1} \equiv \frac{\bar{D}_{3S_1} M_\pi^2}{C_0^{3S_1}} \leq \frac{1}{8.6}, \quad (1)$$

which is more than a factor of two different to what is claimed in [5]. The values of the LECs $C_0^{1S_0}$ and $C_0^{3S_1}$ corresponding to the exponential regulator defined in [6] and the cut-off $\Lambda = 560$ used here and in [1] read:

$$C_0^{1S_0} = -0.143 \times 10^4 \text{ GeV}^{-2}, \quad C_0^{3S_1} = -0.146 \times 10^4 \text{ GeV}^{-2}. \quad (2)$$

The appropriate (not partial wave projected) LECs C_S and C_T , which enters the underlying chiral Lagrangian [6], take the values:

$$C_S = -0.979 F_\pi^{-2}, \quad C_T = -0.003 F_\pi^{-2}, \quad (3)$$

where we have used the same notation (and normalization) as in Ref. [6].^{#4} While the constant C_S is perfectly natural with $\alpha_{C_S} = -0.979$, the LEC C_T appears to be very small as a consequence of the approximate Wigner symmetry, see [6] for more details. The authors of [5] should provide better information on the size of their LECs in the appropriate partial waves. It is now important to address the question whether the range for η_I used in [1] is too narrow? As we have explained there, we have performed detailed and serious studies of the size of LECs appearing at NLO and NNLO in the chiral EFT and *none of the LECs which can be determined from data exceeds ± 3* when properly normalized in units of F_π^2 and Λ_χ^2 . Therefore, the choice for η_I made in [1], which corresponds to $-3 < \alpha_I < 3$, is well rooted in the phenomenology of four-nucleon LECs. Also, if one were to extend this range to say ± 10 , one would leave the realm of

^{#4}Some of the values given here are slightly different from the corresponding ones in [6] due to the different value of $g_{\pi N}$ used, see [1].

a converging EFT because NLO correction would become comparable to the LO contributions. Thus, even as one claims to know nothing about the quark mass dependent LECs, one should still bound them based on such a convergence argument. Finally, as we have discussed in [1], approximate Wigner symmetry requires small values of the corresponding LECs. While such an argument is merely aesthetic, it still should not be discarded completely.

- 2) The central result of Ref. [5] is Fig. 4. This figure as it is shown is more than confusing. First, all other figures are scatter plots, so why are suddenly continuous lines drawn? Simply connecting the most outside lying points by a line^{#5} makes not much sense. In particular, the upper line explodes with decreasing pion masses. A simple extrapolation of this upper line to smaller values of M_π would probably lead to an unnaturally large deuteron binding energy of the order of few hundreds MeV and thus indicates the breakdown of the EFT. This could indicate that the authors have considered a region of parameter space that does not exclude deep bound states outside the range of EFT. The authors of [5] do not clearly discuss the origin of such a dramatic increase of the deuteron binding energy for smaller values of M_π . In Fig. 1, we show our results for the deuteron binding energy taking the range of \bar{d}_{16} as given in the caption of Fig. 4 of [5] and increasing the value of $\eta_{^3S_1}$ given in eq. (1) and used in [1] by a factor of two. The resulting value $\eta_{^3S_1} = 1/4.3$ is even somewhat larger than the one suggested in [5]. The deuteron is bound in the chiral limit and the binding energy only varies between 4 and 20 MeV.^{#6} In our calculation, the expansion of the binding energy is well behaved for all values of M_π . The scattering length in the 3S_1 partial wave calculated with the uncertainties in the LECs as suggested [5] is depicted in Fig. 2.
- 3) There is an apparent inconsistency between Figs. 2,3 and Fig. 4 of [5]. If for pion masses below 60 MeV the deuteron can be bound or unbound, the scattering length in the 3S_1 channel should diverge as the deuteron passes from being bound to being unbound provided the M_π -dependence of the deuteron binding energy is continuous. No such divergence is seen in these figures. We also remark that one rather should plot bands (as done in [1]) because the variations of these parameters cannot simply be treated as statistical fluctuations. Furthermore, as discussed in [4], there are sizeable correlations between certain LECs in that analysis. It is therefore highly questionable to use the MINUIT errors as given in that paper for a statistical error analysis as done in [5].
- 4) It is not clear from Ref. [5] that the calculation is done consistently with the power counting advocated in [2]. In that scheme one should treat the difference between the OPE exchange and its chiral limit representation perturbatively. This is a very slowly converging expansion for *small* energies as demonstrated explicitly in [2]. As shown in Eq. (3) of [5], the full OPE is used in [5] and no uncertainty due to this procedure is given.
- 5) Keeping the short-range terms $\bar{D}_I M_\pi^2$ together with the chiral limit representation of the two-pion exchange (TPE) in the potential as is done in [3], [5] seems to be inconsistent. The reason is that the renormalized expression for the leading chiral two-pion exchange contribution includes apart from the non-polynomial pieces also terms of the kind [1]

$$\alpha_I M_\pi^2 + \beta_I M_\pi^2 \ln \frac{M_\pi}{\Lambda}, \quad (4)$$

^{#5}As was explained to us by one of the authors.

^{#6}We do not consider a variation in \bar{d}_{18} as relevant here. The value we use refers to the well accepted small value of the pion-nucleon coupling constant as deduced by the Nijmegen and VPI/GWU groups. The larger uncertainty due to the fitting procedure in [7] should therefore be taken *cum grano salis*. We will come back to that point later on.

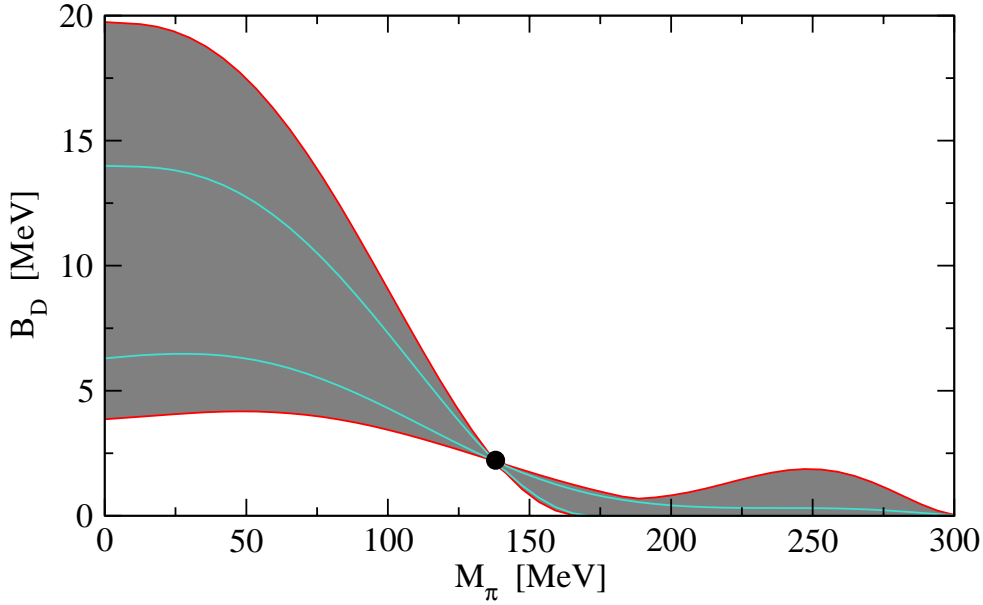


Figure 1: Deuteron binding energy as a function of the pion mass. The shaded areas correspond to the allowed values. The light shaded band refers to the variation of \bar{D}_{3S_1} using $\eta_{3S_1} = 1/4.3$ and $\bar{d}_{16} = -1.23 \text{ GeV}^{-2}$, $\bar{d}_{18} = -0.97 \text{ GeV}^{-2}$. The dark shaded band gives the uncertainty if, in addition to variation of \bar{D}_{3S_1} , the LEC \bar{d}_{16} is varied in the range from $\bar{d}_{16} = -0.17 \text{ GeV}^{-2}$ to $\bar{d}_{16} = -2.61 \text{ GeV}^{-2}$. The heavy dot shows the binding energy for the physical value of the pion mass.

where α_I and β_I are the known constants and Λ refers to the renormalization scale.^{#7} This indicates that the M_π -dependence of the leading TPE is as important as the M_π -dependence of the short-range terms in eq. (4). It is, therefore, not clear, why the authors of [3], [5] decided to neglect the explicit M_π -dependence of the TPE as well as the second term in eq. (4) and to keep only the first term in that equation.

- 6) The authors of [5] claim to be able to reproduce our results using the same input parameters. As we just showed with respect to the discussion of their Fig. 4, we are not able to reproduce theirs. In particular, we obtain for the deuteron binding energy in the chiral limit: $B_D^{\text{CL}} = 9.6^{+4.4}_{-3.2} {}^{+5.7}_{-2.4} \text{ MeV}$, where the uncertainties for \bar{D}_{3S_1} (the first error) and \bar{d}_{16} (the second error) are taken from [5] or are even slightly larger, i.e.: $\eta_{3S_1} = 1/4.3$; $-2.61 \text{ GeV}^{-2} < \bar{d}_{16} < -0.17 \text{ GeV}^{-2}$. As already pointed out, we do not consider a variation in \bar{d}_{18} as relevant here, and, therefore, have not plotted the corresponding bands in Fig. 1. For the sake of completeness, we however calculated the resulting additional uncertainty in the chiral limit value of the deuteron binding energy. Note that the LEC \bar{d}_{18} does not contribute to the OPE in the chiral limit (where the Goldberger-Treiman relation is exact) and thus changing \bar{d}_{18} only affects B_D^{CL} indirectly, due to corresponding small changes in the LECs related to contact interactions, as explained in [1]. Variation of \bar{d}_{18} in the range $-1.54 \text{ GeV}^{-2} < \bar{d}_{18} < -0.51 \text{ GeV}^{-2}$ in addition to variation of

^{#7}Similar terms also arise from renormalization of the short-range interactions by pion loops.

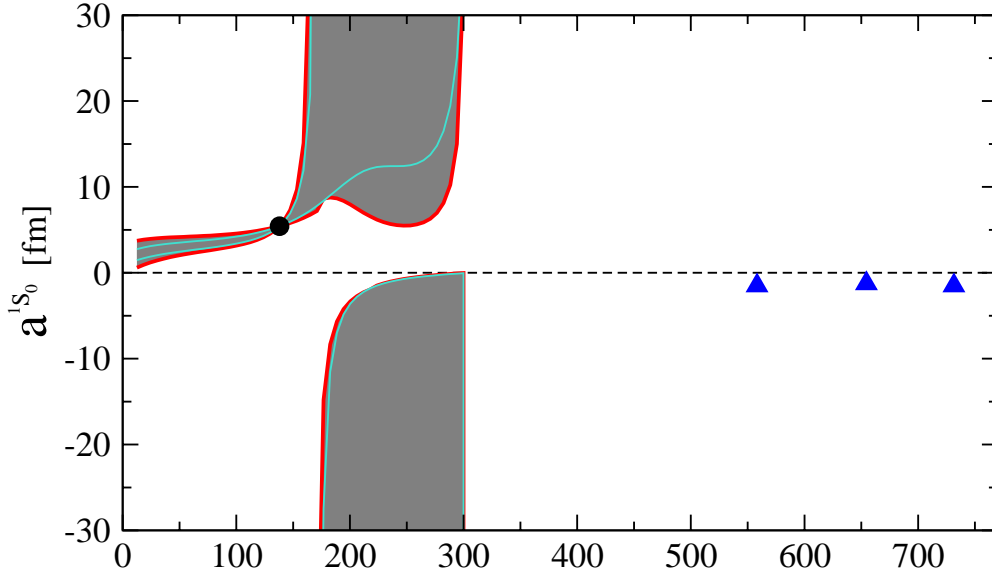


Figure 2: 3S_1 -scattering length as a function of the pion mass. The shaded areas correspond to the allowed values. Bands as in Fig. 1. The heavy dot shows the scattering length for the physical value of the pion mass. The triangles refer to lattice QCD results from [9].

\bar{D}_{3S_1} and \bar{d}_{16} as suggested in [5] leads to $B_D^{\text{CL}} = 9.6^{+4.4}_{-3.2} {}^{+5.7}_{-2.4} {}^{+0.6}_{-0.4}$ MeV. As expected, the resulting additional uncertainty is quite small.

- 7) Some of parameter choices discussed in [5] appear questionable. While all fits of \bar{d}_{16} in [4] give negative values for this LEC, the authors of [5] claim that a positive value of $\bar{d}_{16} = +1 \text{ GeV}^{-2}$ is consistent with the data. It is their duty to present an analysis of the many data on $\pi N \rightarrow \pi\pi N$ using this particular value for \bar{d}_{16} . It should also be stressed that the various fits of $\pi N \rightarrow \pi\pi N$ in [4] are for exactly *one* choice of πN phases and thus *one* value for \bar{d}_{18} (the phase shifts of [8]). Therefore, the independent variation of \bar{d}_{16} and \bar{d}_{18} , the latter referring to other πN phase shift analyses, as done in Ref. [5] does not really make sense.

3. In summary, we have shown that the results presented in [1] are robust under parameter variations, in particular, the deuteron *is bound in the chiral limit*. We have also shown that some of the results presented in [5] cannot be understood simply and thus need clarification. We stress again that the discussions presented here do not influence any of the statements made in our previous work [1], all results and conclusions of that paper remain unchanged.

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